

## Beliefs About What Types of Mechanisms Produce Random Sequences

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### ABSTRACT

Although many researchers use Wagenaar's framework for understanding the factors that people use to determine whether a process is random, the framework has never undergone empirical scrutiny. This paper uses Wagenaar's framework as a starting point and examines the three properties of his framework—independence of events, fixed alternatives, and equiprobability. We find strong evidence to suggest that independence of events is indeed used as a cue toward randomness. Equiprobability has an effect on randomness judgments. However, it appears to work only in a limited role. Fixedness of alternatives is a complex construct that consists of multiple sub-concepts. We find that each of these sub-concepts influences randomness judgments, but that they exert forces in different directions. Stability of outcome ratios increases randomness judgments, while knowledge of outcome ratios decreases randomness judgments. Future directions for development of a functional framework for understanding perceptions of randomness are suggested. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS randomness; equiprobability; fixed alternatives; independence of events

### INTRODUCTION

People's ability to produce and recognize randomness has been an important matter of contention since Reichenbach's (1949) claim that people are incapable of random sequence generation. Since then, numerous studies have supported this claim, and human limitations in randomness production and assessment have been demonstrated within a number of domains (see reviews in Falk & Konold, 1997; Nickerson, 2002; Wagenaar, 1972).

One of the most influential biases in people's conceptions of randomness is a tendency to fall for the "law of small numbers" (Bar-Hillel & Wagenaar, 1991; Budescu, 1987; Falk, 1981; Falk & Konold, 1997; Lopes & Oden, 1987; Tversky & Kahneman, 1971; Wagenaar, 1970a,b, 1972). That is, the balanced distributions typical of lengthy random sequences are believed to apply to smaller samples, resulting in both production and recognition of random sequences being biased toward fewer streaks and greater frequencies of alternation than would be mathematically probable. This common mistake has been implicated as a source of both the gambler's

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fallacy, in which streaks are expected to end with greater-than-chance frequency (Laplace, 1814/1951; Tune, 1964), and the hot-hand effect, in which streaks are expected to continue with greater-than-chance frequency (Gilovich, Vallone, & Tversky, 1985). Noting that the hot-hand and gambler's fallacies are contradicting biases but are typically explained by the same mechanism, many researchers have suggested that one of the factors that influences which bias is exhibited is whether or not the sequence is perceived as having been generated by a random process (Ayton & Fischer, 2004; Burns & Corpus, 2004; for a review see Alter & Oppenheimer, 2006). That is, intuitions about randomness influence predictions about whether a streak will continue or end, and those predictions in turn influence behavior. Further, beliefs about whether or not a sequence was randomly generated influence people's subsequent memory for that sequence—people remember fewer streaks and shorter streaks when they believe a sequence to be random (Olivola & Oppenheimer, submitted for publication). Because perceptions of randomness influence both cognitions and behaviors, it is important to understand people's beliefs about what constitutes a random event.

Psychological studies of conceptions of randomness have focused predominantly on human ability to judge and produce random outcomes, under the assumption that mathematical principles defining random sequences exist and can be used as a comparison standard. However, the mathematical principles used to define “random” outcomes are far from agreed upon and vary widely from study to study. For example, one of the most common definitions of random outcomes is incompressibility—that no formula can be created to describe a sequence that is shorter than the sequence itself (Chaitin, 1975; Kolmogorov, 1965). Such principles are controversial. The decimal expansion of  $\pi$  produces a sequence that has been argued to pass all tests of outcome randomness (Falk & Konold, 1997), and limitless numbers of similar non-randomly produced “random” outcomes have been proposed (Ginsburg & Lesner, 1999). However, some researchers have claimed that the statistical tests that are used for testing randomness are inappropriate (for further discussion, see Ayton, Hunt, & Wright, 1989). Additionally, since any given sequence is equally likely to result from a truly random process, a specific compressible sequence is no less likely to have resulted from a random process than a specific incompressible one. Though principles of random outcomes have been extensively studied, none have emerged as a clearly superior definitive property or even as a reliable indicator.

Given that even experts are unable to agree definitively on what constitutes a randomly produced sequence, it would be far-fetched to suggest that principles of randomness are completely standardized among untrained study participants. Intuitively, however, there must be trends in people's conceptions of randomness, and indeed researchers have investigated what factors influence these conceptions (Wagenaar, 1970a, 1991; see reviews in Falk & Konold, 1997; Nickerson 2002; Wagenaar, 1972).

Attempts to understand the factors that impact perceptions of randomness have primarily investigated features of the sequences themselves rather than the processes that produced these sequences (for a deeper discussion of process vs. outcome analysis, see Zabell, 1992). Some researchers have gone so far as to claim that outcome analysis is the only reasonable way to investigate conceptions of randomness; according to Ginsburg and Lesner (1999, p. 337), “any definition of randomness in terms of process will fail.” However, there is a good deal of evidence that process plays a role in judgments (McDonald & Newell, 2007). Ultimately, any functional framework for understanding perceptions of randomness will have to take both process and outcome into account.

Although avenues for the exploration of randomness in terms of outcome have been extensively studied, very little research has been done on what *processes* are perceived of as random. However, research on process randomness holds great promise for contributing to overall understandings of randomness. The dearth of research in the study of process randomness may stem from circular conceptions of randomness as simply a process that produces unpredictable outcomes (Falk & Konold, 1997; Gell-Mann, 1994). Such studies of process randomness inevitably revert back to analyses of outcome sequences. In contrast, our approach leaves out outcome sequences entirely, allowing us to isolate the principles inherent in the process itself. Although this approach largely ignores the role of outcomes, which we do not dispute are an important part of randomness judgments, it allows for cleaner manipulations and ultimately can provide insight that can be used to improve models that consider both process and outcome.

Very little research has been done on perceptions of randomness that fully isolates the concept of a random process. Lopes (1982) defines process randomness as the unpredictability of an individual event in a series; a construct that Brown (1957) labels primary randomness. Such definitions suggest that randomness is equivalent to unpredictability, which does not clarify the concept of randomness, but merely shifts the predicament to one of defining and measuring aspects of predictability.

The primary framework for understanding process randomness was proposed by Wagenaar (1991). Wagenaar (1991) compiled commonalities among frequently used random generators such as coin flipping, dice tossing, and roulette wheel spinning, and suggests that the common features are potentially definitive properties of random processes. According to Wagenaar (1991, p. 221), the three properties common across frequently used randomness generators are, “a fixed set of alternatives, that remains the same during the generation process,” “a selection of elements that does not utilize previous outcomes,” and “a selection procedure that cannot show a preference for any of the alternatives.” We will refer to these properties more simply as fixed alternatives, independence of events, and equiprobability, respectively. Wagenaar (1991) did not explain why these properties were associated with randomness, nor did he defend them as desirable or normative. Rather, he noted that these three properties seemed to be present across prototypical random processes, and on that basis suggested that they may be central to the notion of process randomness. However, the presence of common features in a system does not imply that those features are causal—many sports occur in the presence of fans, but fans are neither necessary nor sufficient to define sports.

However, the three properties suggested by Wagenaar (1991) were never empirically tested to see whether or not they effectively represent the subjective beliefs held by laypeople about process randomness. They were only suggested as potential and plausible properties. Despite the lack of evidence for these properties actually being objectively or subjectively definitive of random processes (Wagenaar, 1991), they have been used as the standard for random processes, against which the randomness of other generating processes has since been tested (cf. Burns & Corpus, 2004).

The goal of this paper is to empirically test the properties proposed in Wagenaar’s (1991) framework. A set of five studies investigates the nature and direction of the relationships between individual manipulations of these three properties and their effects on perceptions of process randomness.

## STUDY 1A: INDEPENDENCE OF EVENTS

Though dependence of events is typically interpreted to mean dependence of outcomes on previous outcomes, it could also be understood as dependence of outcomes on trial position. We therefore considered both interpretations of the property. Independence based on previous outcome and trial position was systematically manipulated while Wagenaar’s (1991) other properties of randomness were held constant.

### **Method**

#### *Participants*

Participants were 60 adults, recruited from the Princeton University Campus Center. The sample consists primarily of students—both undergraduate and graduate—although occasionally faculty, staff, and visitors not affiliated with the university use the Campus Center as well. Participants were approached individually and recruited to fill out questionnaires in exchange for candy.

#### *Materials and procedure*

Participants were given a questionnaire comprised of a hypothetical scenario involving three variants on a game of dice. In each game, two dice are being rolled one at a time; one die has five blue sides and one red side while the other has five red sides and one blue side. In one variant of the game (Trial Position Dependent), neither die may be rolled twice in a row, so the mostly blue-sided die and mostly red-sided die alternate. In

another variant (Previous Outcome Dependent), if a die lands with a red face up, the mostly red-sided die is rolled next, while if a die lands with a blue face up, the mostly blue-sided die is rolled next. In a final variant (Independent), the dice are both kept in a dark bag and one is blindly selected before each roll. Participants were shown all three conditions and were instructed to rate the randomness of the process of all three of the dice games using nine-point Likert scales (1 = not at all random, 9 = completely random). The order of presentation of the variants was counterbalanced.

### Results and discussion

The overall mean randomness ratings were 7.7 ( $SD = 1.8$ ) for the Independent scenario, 4.9 ( $SD = 2.3$ ) for the Previous Outcome Dependent scenario, and 4.9 ( $SD = 2.7$ ) for the Trial Position Dependent scenario. See Figure 1.

The difference between the Independent scenario and Trial Position Dependent scenario mean randomness ratings was significant,  $t(59) = 6.33$ ,  $p < .01$ , as was the difference between the Independent scenario and the Previous Outcome Dependent scenario mean randomness ratings  $t(59) = 7.51$ ,  $p < .01$ . The mean randomness ratings for the Trial Position Dependent and Previous Outcome Dependent scenarios were not reliably different from one another,  $t(59) = -.04$ ,  $p = .97$ .<sup>1</sup>

The significantly higher randomness ratings in the Independent scenarios indicate that Wagenaar's (1991) property of Independence does indeed appear to be a contributing factor toward the perception of process randomness. However, there was some concern that the results could be driven by the domain in which the study was run (dice). In order to be sure that the results would generalize across different domains, we replicated the study using a scenario involving cards instead of dice.

## STUDY 1B: INDEPENDENCE OF EVENTS

### Method

#### Participants

Participants were 133 students recruited for a questionnaire day via email and posted signs. The questionnaire was embedded in a packet of approximately two-dozen unrelated questionnaires. Participants were paid \$12 for completing the entire packet. Six participants were excluded from the analysis for failing to follow instructions.

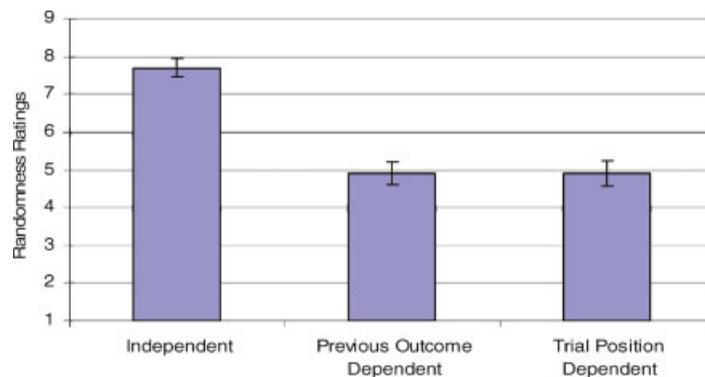


Figure 1. Randomness ratings (1 = not at all random, 9 = completely random) for the Independent (dice drawn from a bag), Previous Outcome Dependent (which die is cast is determined by previous outcome), and Trial Position Dependent (dice are alternated between rolls) versions of the dice scenario

<sup>1</sup>For all of the studies reported in this manuscript, subsequent analyses were done to test for the presence of order effects. However, no reliable (or even close to reliable) order effects were found. Since we had no a priori reason to predict order effects, and none were found, for the sake of brevity we will not describe these analyses in detail.

*Materials and procedure*

Participants were given a questionnaire comprised of a hypothetical scenario involving three variants on a game of cards. In each game, a card is drawn from one of two different decks of cards. One deck of cards has three red suits and one black suit. The other deck of cards has three black suits and one red suit. In one variant of the game (Trial Position Dependent), neither deck may be drawn from twice in a row, so the mostly red-suits deck and the mostly black-suits deck are drawn from alternately. In another variant (Previous Outcome Dependent), if a red card is drawn, the mostly red-suits deck is drawn from next, while if a black card is drawn, the mostly black-suits deck is drawn from next. In a final variant (Independent), one deck is blindly selected to be presented, and a card is drawn from whichever deck is chosen. Participants were shown all three conditions and were instructed to rate the randomness of the process of all three-card games using nine-point Likert scales (1 = not at all random, 9 = completely random). The order of presentation of the game variants was counterbalanced.

**Results**

The overall mean randomness ratings were 6.8 ( $SD = 2.2$ ) for the Independent scenario, 3.7 ( $SD = 2.2$ ) for the Previous Outcome Dependent scenario, and 5.1 ( $SD = 2.5$ ) for the Trial Position Dependent scenario. See Figure 2.

The difference between the Independent scenario and Trial Position Dependent scenario mean randomness ratings was significant,  $t(126) = 6.50$ ,  $p < .01$ , as was the difference between the Independent scenario and the Previous Outcome Dependent scenario mean randomness ratings  $t(126) = 12.27$ ,  $p < .01$ . The mean randomness ratings for the Trial Position Dependent and Previous Outcome Dependent scenarios were also significantly different,  $t(126) = 5.43$ ,  $p < .01$ .

**Discussion**

The Independent scenarios in both Studies 1a and 1b received higher randomness ratings than either of the Dependent conditions. This suggests that Wagenaar's property of Independence is indeed a contributing factor toward the perception of process randomness. It is interesting to note that despite the fact that the Trial Position Dependent and Previous Outcome Dependent scenarios violate the property of independence of events in very different manners, they received effectively the same ratings of randomness in Study 1a. In Study 1b, however, participants found the Trial Position Dependent scenario to be more random than the

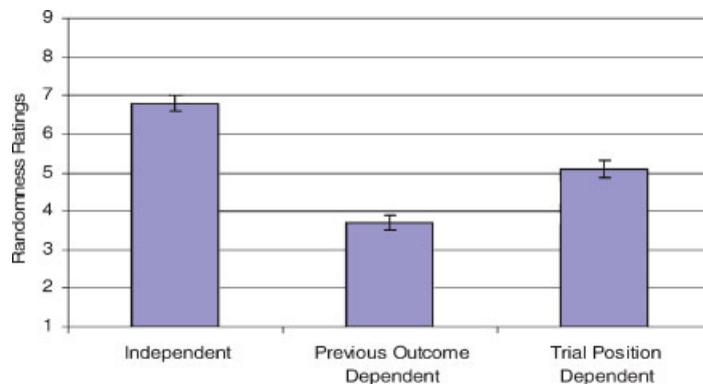


Figure 2. Randomness ratings (1 = not at all random, 9 = completely random) for the Independent (deck chosen from behind a dealer's back), Previous Outcome Dependent (deck is determined by the previous outcome), and Trial Position Dependent (decks alternate between draws) versions of the card game scenario

Previous Outcome Dependent scenario. This suggests that while the mere existence of any form of dependency is enough to lower perceptions of randomness, the extent to which this occurs may depend on the manner in which the dependency is created. It is worth noting that Outcome Dependency is a more faithful interpretation of Wagenaar's (1991, p. 221, emphasis added) original principle that random processes possess a "selection of elements that does not utilize previous *outcomes*." It is therefore somewhat surprising that the increased randomness judgments for Outcome over Trial Dependency were only found in the domain of cards, and not found in one of the domains from which Wagenaar derived his properties of randomness (dice). It is unclear why these domains should differ in this way.

What is clear, and arguably more important, however, are the significantly higher ratings of the Independent scenario's randomness over that of the two Dependent scenarios in both Studies 1a and 1b. These differences indicate a generally positive relationship between the independence of events and perception of randomness in processes. The question of how different types of dependence influence judgments of process randomness, and how those influences may vary based on domain, is still an open matter for future investigation.

## STUDY 2: FIXED (KNOWN) ALTERNATIVES

The property of fixed alternatives is that random processes have "a fixed set of alternatives, that remains the same during the generation process" (Wagenaar, 1991, p. 221). The concept of fixed alternatives, as proposed by Wagenaar (1991), could have referred to fixedness of ratios of meaningful outcomes, fixedness of total numbers of outcomes, or both. For instance, when drawing colored balls from an opaque container, one might or might not know what percentage of the balls are of each color, and one might or might not know the total numbers of balls in the urn. Since it is unclear whether the property of fixed alternatives refers to ratio or number, we considered both interpretations of the property.

### Method

#### *Participants*

Forty-two adults were recruited from the Princeton University Campus Center. The sample consists primarily of students—both undergraduate and graduate—although occasionally faculty, staff, and visitors not affiliated with the university use the Campus Center as well. Participants were approached individually and recruited to fill out questionnaires in exchange for candy.

#### *Materials and procedure*

Participants were given a questionnaire, composed of six variants of a hypothetical game. The game consisted of one opportunity to reach into an urn while blindfolded to pull out a colored ball, where the different colors corresponded to different prizes.

The six scenarios differed in regard to the degree of knowledge about the total number of balls in the urn, and the ratio of the colors of those balls. A two-by-three factor within-subjects design was used to present all participants with six different versions of the same basic selection scenario such that there were two levels of knowledge about total numbers of outcomes and three levels of knowledge about ratios of meaningful outcomes. In the Known Total (KT) scenarios, participants were told that there were 100 balls in the urn; in the Unknown Total (UT) scenarios, the total number of balls in the urn was not specified. In the Known Ratio (KR) scenarios, participants were told that half the balls were red and half were green; in the Partially known Ratio (PR) scenarios, participants were told that at least one ball was red and at least one was green; in the completely Unknown Ratio (UR) scenarios, nothing was known about the color distribution of the balls. The

ordering of the scenarios was counterbalanced such that half the participants saw the scenarios in one order (KT/UR, KT/PR, UT/PR, KT/KR, UT/KR, UT/UR), and half saw the scenarios in the reverse order: (UT/UR, UT/KR, KT/KR, UT/PR, KT/PR, KT/UR).

Participants were instructed to rate the randomness of the process on a one to nine scale where one was not at all random and nine was entirely random.

## Results

The results are displayed graphically in Figure 3. On average, participants rated the variants of the game with unknown ratios of outcomes ( $M = 7.54$ ) as more random than those with partially known ratios ( $M = 6.30$ ), which in turn were seen as more random than fully specified ratios ( $M = 5.27$ ). A factorial ANOVA confirmed that these differences were reliable  $F(2, 246) = 21.30, p < .01$ . Meanwhile, there was no observable difference between variants of the game for which the total number of balls was known ( $M = 6.37$ ) and those for which the total number of balls was not ( $M = 6.37$ ), nor was there a reliable interaction.

## Discussion

When the concept of fixed alternatives is interpreted as a ratio of alternatives instead of total number of alternatives, it seems that greater fixedness of alternatives leads to *less* random perceptions of the process. This suggests that lack of knowledge, or ambiguity might be one source of perceptions of randomness. Thus, while it is true that fixedness of alternatives plays a role in randomness judgments, it initially appears to do so in the opposite direction from what Wagenaar (1991) had hypothesized.

It is important to note that in Study 2 the term “fixed” was interpreted to mean “known.” However, “fixed” could also be interpreted to mean stable or unchanging. Indeed, part of Wagenaar’s (1991, p. 221) discussion of fixed alternatives was that the set of alternatives “remains the same during the generation process.” Study 3 was run to examine whether the stability of options over time has an effect on perceptions of process randomness.

### STUDY 3: FIXED (UNCHANGING) ALTERNATIVES

## Method

### Participants

Participants were 38 adults recruited from the Princeton Junction Train Station during the morning commute, a population primarily consisting of middle class adults. Participants were approached individually and recruited to fill out questionnaires. Participants were not compensated for their time.

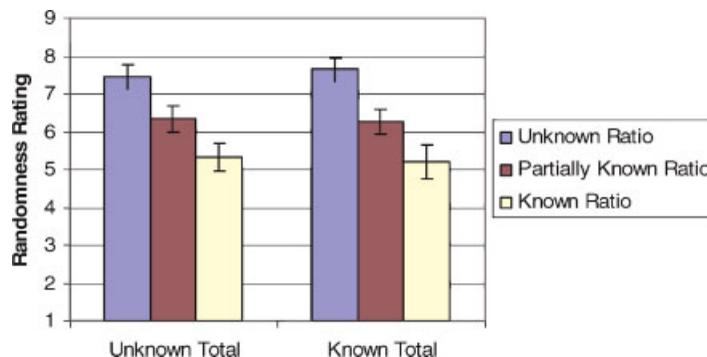


Figure 3. Randomness ratings (1 = not at all random, 9 = completely random) of six different scenarios of balls being pulled from an urn which vary on how much information was given about the total number of balls, and the ratio of ball colors

### Materials and procedure

A hypothetical scenario was developed involving a hypothetical tourist attraction: The World's Biggest Gumball Machine, which holds 10 000 gumballs. Each day, 1000 visitors use the machine and receive a gumball. These gumballs are then replenished by a nearby gumball factory. Participants were asked to rate the randomness of the process of receiving a gumball given three different mechanisms for replenishing the machine's stock. In the Fixed condition, every day the gumball factory makes all of its varieties of gumballs and replenishes the machine with the different gumball flavors in the same ratio. In this way, the number and ratio of alternatives remains approximately constant from day to day. In the Non-Fixed condition, every day the factory chooses a single flavor of gumball to make and they use that one flavor to replenish the machine. Since on a given day, all 1000 gumballs added to the machine are of the same flavor, the ratio of flavors in the machine changes based on the day. In the Ambiguous condition, everything about gumball production is kept a secret. The ratio of different flavors produced each day is unknown. Therefore, it is unknown whether the ratio of flavors in the machine varies over time. Each participant was shown all three conditions. In each case, the participant was asked to rate the randomness of the process of *receiving a gumball* given the different ways that the machine was stocked using nine-point Likert scales (1 = not at all random, 9 = completely random). The order of presentation of the scenarios was counterbalanced.

### Results

The overall mean randomness ratings were 5.7 ( $SD = 2.7$ ) for the Fixed scenario, 3.6 ( $SD = 2.0$ ) for the Non-Fixed scenario, and 3.9 ( $SD = 2.1$ ) for the Ambiguous scenario. See Figure 4.

Paired sample *t*-tests show that the difference between the Fixed scenario and Non-Fixed scenario randomness ratings was significant,  $t(37) = 3.73$ ,  $p < .01$ , as was the difference between the Fixed scenario and the Ambiguous scenario mean randomness ratings  $t(37) = 3.04$ ,  $p < .01$ . The mean randomness ratings for Non-Fixed scenario and the Ambiguous scenario were not reliably different from one another,  $t(39) = .484$ ,  $p > .10$ .

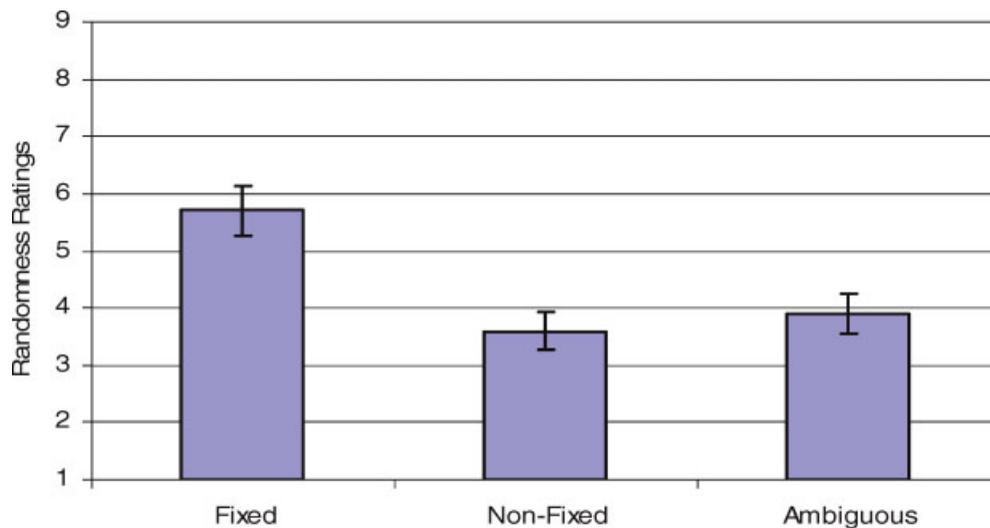


Figure 4. Randomness ratings (1 = not at all random, 9 = completely random) for three different versions of the gumball factory scenario which vary on how much information was given about the ratio of gumballs over time

### Discussion

In contrast with the results of Study 2, in Study 3 participants judged processes with fixed alternatives to be more random than those without fixed alternatives. However, this discrepancy can be explained by differences in how the term “fixed” was defined. In Study 2, “fixed” referred to whether ratios of outcomes were known, whereas in Study 3, “fixed” referred to whether the ratios of outcomes were stable over time. Since Wagenaar (1991, p. 221) did explicitly refer to a distribution of outcomes that “stays the same during the generation process,” it is reasonable to presume the latter definition is the one he would endorse. As such, this Study supports his second property.

A critic might argue that Study 3 is actually manipulating equiprobability rather than fixedness. While the probabilities are not actually stated in any of the conditions, the non-fixed condition must deviate from equiprobable by definition; maintaining equiprobable ratios across trials requires the ratios to be fixed. Meanwhile, participants may have assumed equiprobability in the other conditions, in the absence of explicit evidence to the contrary. We believe this is an unlikely explanation for the results primarily because the ambiguous condition more closely resembles the non-fixed than the fixed condition. The fixed condition does not imply equiprobability any more than the ambiguous condition. Were this alternative explanation true, one would expect those two conditions to look more similar. However, it is worth noting that this definition of fixed alternatives is, in a way, inherently conflated with equiprobability. It is impossible to be equiprobable if the alternatives are not fixed, which means that in many cases these two principles will be redundant.

In Study 4, however, we held fixedness constant while manipulating equiprobability in an attempt to validate the use of equiprobability as a cue for process randomness.

## STUDY 4: EQUIPROBABILITY

Wagenaar (1991, p. 221) defines selection procedures with equiprobability as those that “cannot show a preference for any of the alternatives.” Equiprobability was systematically manipulated by comparing processes that were equally likely to produce each possible outcome to those for which some outcomes were more likely than others.

### Method

#### *Participants*

Participants were 120 adults, recruited from the Princeton University Campus Center and the Princeton Junction Train Station. The demographics of the population were the same as in Study 1. Participants were approached individually and recruited to fill out questionnaires. At the Campus Center they were paid for their time with candy, at the train station they were not reimbursed. One participant’s results were discarded for failing to comply with instructions.

#### *Materials and procedure*

Participants were given a questionnaire that asked them to imagine a store’s promotional giveaway, in which all customers who spend over \$50 are allowed a single spin on a wheel that determines the gift that the customer receives (either a CD or DVD). The questionnaire consisted of two scenarios for the promotional wheel, such that one wheel contained an equiprobable distribution of alternatives, and the other contained a non-equiprobable distribution.

Participants were instructed to choose which version involved the more random process. As a precautionary measure, we also offered the participants the option to rate the two wheels as equally random. This was meant to ensure that any observed effects would not result merely from forced choice.

One ambiguity in Wagenaar's (1991) framework is whether equiprobability should be viewed as a binary construct or if equiprobability can be viewed as a matter of degree. Extending equiprobability to a non-binary framework, it would be expected that if the property held true, then the further from equiprobability a scenario was, the less random it would seem. To test this possibility, we included a between-subjects manipulation of the non-equiprobable wheel, such that half the subjects compared the 50:50 wheel to a 99:1 wheel, and half compared the 50:50 wheel to a 60:40 wheel. If the property of equiprobability influences judgments in a continuous fashion, we would expect a higher frequency of random rankings for the 50:50 wheel in the 99:1 comparison condition than in the 60:40 comparison condition.

The order of the wheels' presentation was counterbalanced as was the presentation of the CD or DVD as the odds-favored gift within the non-equiprobable conditions.

## Results

The distribution of rankings is shown in Table 1. The most common response was to rate the equiprobable and non-equiprobable options as equally random ( $n = 52$ ). However, considering only participants who had a preference, people were more likely to judge the equiprobable option as more random ( $n = 45$ ) than to judge the non-equiprobable option as more random ( $n = 23$ ), and a comparison against the null hypothesis of a 50:50 split was reliable,  $\chi^2(1, n = 68) = 7.11, p < .05$ . Using a 99:1 non-equiprobable comparison wheel as opposed to a 60:40 non-equiprobable comparison wheel led to no reliable differences in ratings in a  $2 \times 2$  chi-square,  $\chi^2(2, N = 68) = .59, p > .10$ .

## Discussion

The property of equiprobability leads to the prediction that participants would rank the 50:50 wheel scenarios as more random than the non-equiprobable wheels, and this was indeed the case. Further, this experiment was somewhat conservative in that it allowed participants to rate the options as equally random. As such, the results are unlikely to be an artifact of a forced choice procedure.

One possible extension of equiprobability was the notion that the farther that a distribution of outcomes strayed from equiprobability, the less random the process would seem. This was not the case. Moving a process further from equiprobability had no observable effect.

A critic might argue that our operationalization of equiprobability was flawed in this study. While we considered the scenario non-equiprobable because the spinner has a higher probability of landing in one area than another, this contains the implicit assumption that the spinner is equally likely to land on any given point in the circle. That is, whether the process is equiprobable depends on whether the focal outcome is the exact point where the spinner lands on the wheel, or whether the spinner lands in the CD area as opposed to the DVD area. However, given that the emphasis of the scenario was on winning a CD versus DVD, we believe that this distinction was more focal for the participants. This interpretation is supported by the fact that there were observable differences between the equiprobable and non-equiprobable conditions; if

Table 1. The number of participants choosing either equiprobable or non-equiprobable ratios of outcomes to be more random for both 60:40 and 99:1 operationalizations of non-equiprobable

	60:40 Comparison condition	99:1 Comparison condition
Equiprobable most random	21	24
Equally random	26	26
Non-equiprobable most random	13	10

participants had focused on the exact location of the spinner, then both conditions would have been equiprobable and no differences would have been observed.<sup>2</sup> However, this criticism does leave open questions of how people determine which elements of a process to focus on and what they consider an outcome to be, both of which will ultimately be important for a complete understanding of perceptions of randomness.

## GENERAL DISCUSSION AND CONCLUSION

Wagenaar's (1991) three properties are a prominent framework in understanding process randomness (cf. Burns & Corpus, 2004), but had previously not been experimentally validated. In this paper, we showed that all three of the properties that he identified do indeed play a role in judgments of randomness. However, the properties are suitably vague such that each could be interpreted in several ways. Exploring the different interpretations of Wagenaar's properties has helped delineate our understanding of randomness judgments, while also creating new areas for exploration.

Independence of events does seem to correspond to greater perceptions of process randomness regardless of how it is defined (Studies 1a and 1b). However, events can be dependent in various ways—for example, dependent on trial position, or dependent on previous outcome. We found mixed results on the extent to which these different types of dependencies influence perceptions of randomness. There may very well be an interaction between the type of dependency, and the domain in which randomness is being assessed (e.g., cards vs. dice). As such, the exact nature of how independence influences judgments of randomness is complex, and not fully elucidated.

The property of Fixed Alternatives also has an effect on randomness regardless of how it is instantiated (Studies 2 and 3). However, depending on how it is defined, it can either increase or decrease perceptions of randomness. Defining “fixed” as stable over time leads to increased perceptions of randomness, as Wagenaar (1991) predicted. However, defining “fixed” as known leads to decreases in perceptions of randomness. This implies that these may actually be two separate properties, each of which has an influence on randomness judgments. In other words, we have identified a fourth property of process randomness that has not previously been well established in the literature. It is also worth noting that these two “fixedness” properties may interact in interesting ways. Knowledge of ratios decreases randomness ratings, whereas stability of ratios or numbers over time actually increases randomness ratings. This is an area ripe for further exploration.

The property of Equiprobability is also a predictor of randomness ratings (Study 4). Processes that lead to equiprobable outcomes are judged as more random than those that favor a particular outcome. Surprisingly, though, this appears to be binary in nature. Increasing the deviation from equiprobable had no observable effect. While it is always difficult to argue from a null result, the large sample size (120 participants) coupled with the extreme non-equiprobable option (99:1) make it unlikely that this was due solely to a lack of power. However, future research might consider more sensitive measures such as a within-subject design to further explore this issue.

This result is also challenging for other major frameworks in perceptions of randomness as it calls into question the validity of the idea that process randomness is purely a measure of outcome unpredictability (Brown, 1957; Lopes, 1982). After all, the more equiprobable a set of potential outcomes, the less predictable an individual result. The results of a process with a 99:1 outcome distribution are much more predictable than the results of a process with a 60:40 outcome distribution. Since relative equiprobability does not reliably

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<sup>2</sup>This could, to some degree, account for the large proportion of participants who rated the two outcomes as equally random. However, this is not a confound, just an addition of noise. Those participants who believed both scenarios to be equiprobable would not be biased in either direction, and as such they would either choose the “equally random” option, or would choose randomly and their selections would cancel each other out.

increase the perception of randomness, and equiprobability affects unpredictability, unpredictability cannot be the primary driver of perceptions of randomness when participants are asked to focus on the randomness of the process instead of randomness of the outcome. Similarly, in Study 3, the more knowledge one had, the less random a scenario was rated, which suggests that unpredictability on its own is not a sufficient definition of process randomness.

These findings can also help inform Bayesian models of human randomness judgments (e.g., Griffiths & Tenenbaum, 2003; Tenenbaum & Griffiths, 2001). Bayesian models are extremely promising for the understanding of perceptions of randomness because they take both process and outcome into account. These models posit that when people are asked the question, “what are the odds the process randomly generates outcomes, given the outcome” they consider the odds of receiving a particular sequence given a random versus nonrandom generating process (likelihood ratio), and the prior odds of the process being random versus nonrandom (priors). One challenge facing such models is that they make very different predictions depending on how one sets the priors. By better understanding what factors lead people to have priors that a process is random, Bayesian approaches will be able to more accurately model perceptions of randomness.

Further, Bayesian approaches may be able to better explicate the nature of process randomness. In these studies, we deliberately ignored the role of outcomes in order to better isolate people’s preconceived notions of what features of a generating process lead to randomness. However, outcomes are clearly important, and may interact with perceptions about process in interesting ways. For example, perceptions about randomness of a process being used to generate a single event may differ from beliefs about that same process if it is used to generate multiple events. This interaction between process and outcome could potentially be modeled effectively using a Bayesian updating process.

Though these studies have validated Wagenaar’s (1991) three properties, and identified an additional property in the form of lack of knowledge, there may also be a number of other factors that may play a role in judgments of process randomness. For example, the intentionality of a given outcome may make a process seem less random (cf. Caruso & Epley, 2004). Regardless of the actual controllability of a given outcome, it is possible that having desired or intended outcomes—for instance, a free throw shooter intending to make a shot or a gambler intending to roll a six—may lead to perceptions of processes being less random. Another potential factor is non-determinism. It is possible that predetermined outcomes may be perceived as less random than those determined on the spot, even if the predetermined outcomes remain unknown until they are revealed. For example, imagine if a lottery number was selected by taking the number of seconds left until midnight ( $n$ ), and finding the  $n$ th digit of  $\pi$ . Although nobody knows the exact number of seconds until midnight, nor what number that digit of  $\pi$  would yield, the process may be perceived as nonrandom because it is determined by a known algorithm.

Yet another possible influence comes from the developmental theory set forth by Piaget and Inhelder (1951). According to Piaget and Inhelder (1951, p. 222), development of a full concept of chance and randomness involves acquiring concrete and formal operations, and passing through stages of failing to differentiate the necessary and the possible, discovering chance “as an antithesis of operations,” and finally, despite the inherent role of chance, seeking to impose order on random events. Future studies might test the role of development in perceptions of randomness—if children perceive random processes differently than adults, it might yield insight into the mechanisms by which chance is understood. Additionally, the metacognitive and experiential correlates of events such as processing fluency could impact the perception of randomness (e.g., Schwarz, 2004).

In searching for a framework for understanding perceptions of randomness, it is important to remember that ultimately, there may not exist a definition of true, binary, objective randomness that everyone will agree upon. Nonetheless, because perceptions of randomness influence beliefs (Burns & Corpus, 2004), behaviors (Ayton & Fischer, 2004), and memory (Olivola & Oppenheimer, submitted for publication), our understanding of cognition will be enhanced by a deeper understanding of how randomness is perceived. In seeking to understand the role of the relationship between observer and process as a determinant of

perceptions of randomness, psychologists can progress toward the development of a functional framework of the perception of process randomness, and toward a more complete understanding of the perception of randomness more generally.

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