

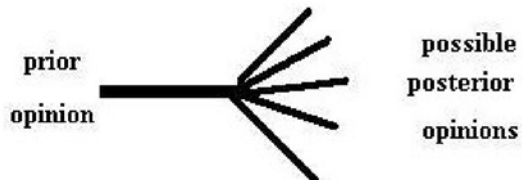
## PHI 533 Decision Theory

### Session 1: Updating

September 18, 2002

[*preface with note re:* [http://www.princeton.edu/~bayesway/Book\\*.pdf](http://www.princeton.edu/~bayesway/Book*.pdf)]

Additional reference: . "Symmetries in Personal Probability Kinematics" in N. Rescher (ed.) *Scientific Inquiry in Philosophical Perspective*. Lanham, MD: University Press of America, 1987, pp. 183-224.



- Tasks:**
- 1) to describe states of opinion
  - 2) to describe possible inputs
  - 3) to describe the ways of going from prior opinion and input to posterior opinion
  - 4) and in all three of the above, to explore criteria for assessing/evaluating them

### Simple Belief model

three-fold evaluative epistemic/doxastic attitude toward propositions, with propositions as input ['ticker tape model']

Basic imperative (synchronic): be consistent!

Are there any diachronic criteria that do not derive from this? [NO: G. Harman, *Change in View*. YES: orthodoxy. YES & NO: normal vs revolutionary epochs]

### Simple Subjective Probability model

Vague opinion with nuances: qualitative, comparative, quantitative

Basic imperative (synchronic): be coherent!

**candidate for coherence:** compatibility with a probability function

Muddy Venn diagram to explain probability and conditional probability

*Input:* a proposition

*Possible responses:*

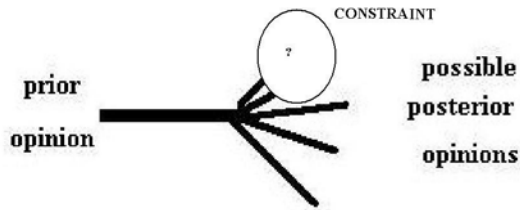
- give probability 1 ('revelation model'; experience speaks with the voice of an angel) **Simple Conditionalization** [SC] [Muddy Venn Diagram applied]
- give probability  $x < 1$  ('lying spies model') **Jeffrey Conditionalization** [JC] [Muddy Venn diagrams combined] [Some notes on *mixtures* of probabilities]

### A Different View of Inputs

Models of input: a) passive, ticker tape, revelation model

b) robot model [Hartry Field]

c) voluntarist model: in response to experience the person accepts a constraint on what the posterior opinion can be.

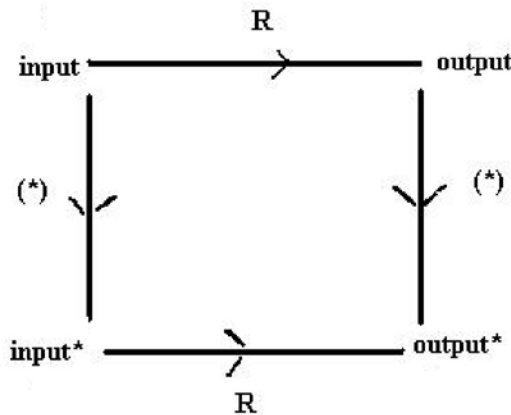


**Question: Is There Any Justification or Rationale for Such Rules as SC, JC?**

- 1) betting and trading rationales ('Dutch Book arguments') -- not taken up now
- 2) prevision of one's own future opinions -- to be taken up next time
- 3) 'minimize change' [doxastic conservatism] ... we will look at some below
- 4) symmetry arguments

**Symmetry Arguments in general**

The general idea is that a problem can be solved by first transforming it into an "essentially the same" problem, which is easier to solve, and then translating back again to the original.



The crucial information needed is: what counts as "essentially the same"? Or, to keep the matter connected with the specific problem context, as "relevantly the same", "the same with respect to all relevant factors" (relevant to .... ?)

Suppose the relevant factors are F, G, H, ... Then that means that the output has to be a *function of (depend solely on)* those factors.

**Application to SC and JC**

**SC:** To find a rule that transforms the prior opinion into a posterior opinion which gives probability 1 to the input proposition.

Assumption: that the prior [ i.e.prior probabilities of propositions in the prior's domain] and the input proposition are the sole relevant factors.

**JC:** To find a rule that transforms the prior opinion into a posterior opinion which gives probability  $x$  to the input proposition.

Assumption: that the prior [ i.e. prior probabilities of propositions in the prior's domain] and the input proposition are the sole relevant factors.

### Intuitive explanation of the proof for SC

The important thing to notice is that if ratios of probabilities inside  $A$  stay the same, then we are done. [DISCUSS]

For a simple case, suppose the input proposition is  $A$  and that this is true in just 5 worlds, which receive equal prior probabilities. Then they will receive equal posterior probabilities. For how could a function that depends only on  $A$  and the prior treat one of those worlds differently -- given simply that they are parts of  $A$  that were treated the same by the prior?

Suppose now that the 5 worlds had different prior probabilities, but all those probabilities were rational fractions, for example  $3/5$ ,  $1/5$ ,  $1/15$ ,  $1/15$ ,  $1/15$ . Then we can replace the first world by 9 worlds, and the second by 3 worlds, with all the worlds now receiving  $1/15$ . That does not change the problem essentially: the new worlds are different in ways that are irrelevant to the original problem so no wonder they were not distinguished ... :-) Now we have reduced the case to the previous problem (for the number was 5 is not essential to that reasoning).

For the arbitrary case we'll replace worlds by non-denumerable sets, with appropriate measures on them -- i.e. heaps of very fine mud -- and repeat the argument using calculus rather than arithmetic. See *Laws and Symmetry* pp. 335-337.

### **Generalized Probability Kinematics**

Jeffrey already generalized this to constraint "For partition  $A, B, C, \dots$ , let the probabilities be  $x, y, z, \dots$  (summing to 1)". [JC in general form]

But what if  $A, B, C, \dots$  are not a partition, but overlap each other? And what of different constraints altogether, like "For propositions  $A, B$  let the conditional probability of  $B$  given  $A$  be  $x$ "? [Still further forms: constraints on expectation values, correlation]

### **The Judy Benjamin Problem**

That is the simplest "next" possible constraint. No symmetry argument appears available. The conservative 'minimize change' imperative appears to be ambiguous, and there are three versions:

The minimal mud change rule [G. Harman]

The projection rule [R.I. G. Hughes]

The minimize relative information rule ['INFOMIN', 'MAXENT', 'MRI': Jaynes]  
(I will explain only the first one, but give some intuitive idea of the other two)

### Assessing proposed rules for updating opinion

Isaac Levi's complaint (put in our terms): the transition from the morning's prior to the evening's posterior opinion should be independent of the order in which the constraints were accepted. This is so for SC but not so for Jeffrey Conditionalization.

**Question:** if a person uses the given rule a number of times on 'good' inputs, with sufficient repetition, will his/her posterior converge to a limit that is independent of the order? [A demonstration of the affirmative is a "Swamping theorem" --proved for **JC**]

Simulated horse races: there are  $N$  horses in the race, but the viewer gets to see two of them at a time, and revises his odds for those two each time. As it happens, the revision is always to what will turn out to be the final odds posted by the bookie. If the viewings are repeated ad infinitum, how soon -- if ever! -- do we see an effective convergence to the final odds? (Faster convergence is better.)

The Wavering Effect. Suppose the viewer keeps seeing the same pair of horses in different circumstances, and wavers e.g. between 2:1 and 1:2 -- what will happen? Application to the Judy Benjamin problem.

### CONCLUSION

Generalized probability kinematics is the study of *normal, rule governed* updating in response to accepted constraints of many different forms. Symmetry arguments are so far available for the basic rules. There are other putatively a priori arguments in the literature for the INFOMIN rule, but also arguments for its being essentially a rule of thumb of limited value. (References available ...) All the rules, however, require for their application quite a few choices: how to represent the total possibility space, and what to count as factors relevant to the optimal satisfaction of the constraint. (Think of the Monty Hall problem, 'three prisoners' problem, etc.)

Most of all, this study is based on the assumption that there must be a uniformly applicable rule as solution to the problem -- 'rational implies rule-governed' you might say. Next time we will set that assumption aside.